COMMUNICATIONS TO THE EDITOR

Fins in Series

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For the purposes of this discussion, a fin is defined as a body of thermal interest in which the temperature differences in the cross section are unimportant compared with those in the

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A Note on Mixing Rates in Stirred Tanks

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The purpose of this note is to present a comparison of the recent data of Biggs (*4*) with the mixing time correlation of Landau and Prochazka (*8*) and to indicate a distinction between the terminal mixing time of Biggs and of Norwood and Metzner (*10*).

Landau and Prochazka presented a model of the homogenation process in a batch baffled turbulent stirred tank. The model left unresolved three parameters. These parameters were determined by following the rate of batch homogenation of miscible liquids in baffled stirred tanks in the turbulent region ($N_{Re} > 10^4$) with conductometric, thermometric, and colorimetric probes (*7*). Mixing time correlations were developed for three blade square pitch propellers, six blade fan disk turbines, and four blade 45 deg. inclined paddles. Their correlations are:

Propeller

$$nt = 3.48\ (D/d)^{2.02} \log\ (2.0/X) \tag{1}$$

45-deg. paddle

$$nt = 2.02\ (D/d)^{2.20} \log\ (2.0/X) \tag{2}$$

Fan disk turbine

$$nt = 0.905 (D/d)^{2.57} \log\ (2.0/X) \tag{3}$$

The quantity X is the approach of the entire tank contents to homogeneity. At any time X is the integral mean value throughout the tank of the local degree of homogeneity which is defined as

$$x = \frac{|C(t) - C(f)|}{C(f) - C(i)} \tag{4}$$

Landau and Prochazka found the mean value of X at time zero to be 2.0.

Biggs introduced a salt pulse into the feed stream of a flow stirred tank and measured several time quantities of the exit stream response curve. The time for the exit stream to settle to ±5% of its quasi steady value (Biggs-t_3) will be considered here and will be designated $t(0.05)$.

Certain differences exist between the test conditions of the two studies; they are presented in Table 1.

In both systems the liquid depth was equal to the tank diameter, and the mixing tank contained four wall baf-

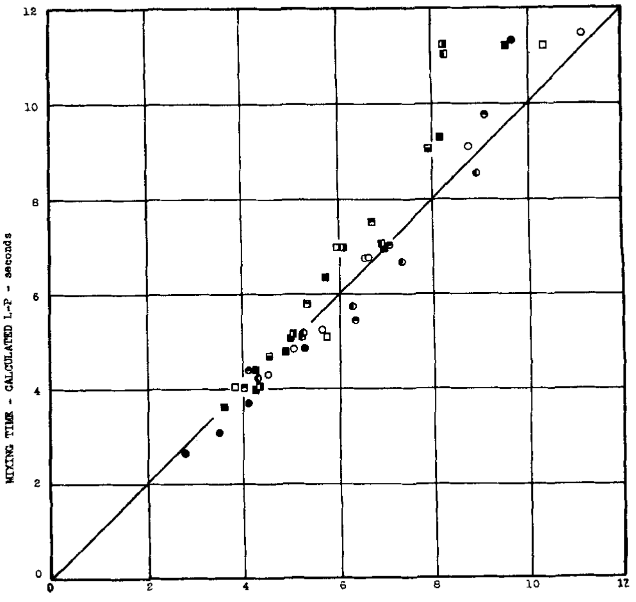

Fig. 1. Overall comparison of Biggs' data with Landau-Prochazka correlations.

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axial direction x.

It is often desirable to calculate the temperature distribution in a complex structure formed by joining members having good thermal conductivity. Frequently, such a structure will resemble a system of fins connected in series. In order to take advantage of this similarity, analytical expressions have been derived and are given below for two, three, and four fins in series. Each fin may have its own surface conditions, axial conductance, and internal heat generation. In addition, there can be thermal resistances and/or heat losses between fins.

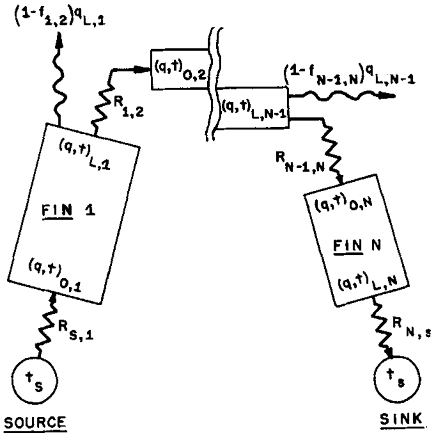

Fig. 1. Illustration of nomenclature.

The nomenclature used is illustrated in Figure 1. There, fin N is located in opposition to fin 1 to emphasize that even heat transfer between unconnected fins may be included, if a little ingenuity is exercised. (In this regard, the section on Plate Fin with Different Conditions at Each Surface is relevant.) Thus, useful solutions can be found for a large variety of problems which would normally justify the use of extensive computational equipment.

BASIC EQUATIONS

As shown by Jakob (*1*), the temperature distribution in a fin may be expressed as

$$\theta = Me^{-mx} + Ne^{mx} + [q'''/(km^2)] \tag{1}$$

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certainly be more exact than the graphical procedures described. In fact, the failure of the author to describe numerical procedures is the main fault in this book.

In general, I would say that this book is a valuable addition to the library of any student of thermodynamics mainly because of the thoroughness with which it treats the classical theories of solutions. Because it is so cursory in its treatment of fundamentals, it must be recommended as a supplement to a standard text, rather than as a replacement. Its value to researchers in the field is more limited because it is simply too short to treat modern advances with the thoroughness they deserve.

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Equilibrium Thermodynamics, J. Coull and E. B. Stuart, Wiley, New York (1964). 469 pages.

The presentation of classical thermodynamics given by the authors differs from the normal presentation in three respects. First, to quote the authors, "Our method of presentation of material is based on the spiral technique wherein the various topics are presented early in the book and are developed in depth as the student progresses." Second, "Detailed references to equipment have been largely omitted, for we feel that the engineering application of equilibrium thermodynamics should be at an analytical level. We have brought in details of application not in equipment, but in the determination of thermodynamic properties and equilibrium analysis." Third, the authors have included a chapter on the third law, which contains a brief discussion of microscopic considerations, and an introductory chapter on nonequilibrium thermodynamics.

These last two points are certainly noteworthy. It is refreshing to encounter a complete chapter (Chapter 6, 35 pages) entitled "The Evaluation of the Thermodynamic Properties from Fundamental Data." The authors clearly inform the reader of the purpose of the chapter, and of the desirability of making such calculations. This section is well illustrated with examples and includes the standard methods of calculation from P-V-T data, and also a discussion of generalized properties. Although the presentations of microscopic considerations and nonequilibrium thermodynamics are admittedly brief, the former is sufficient to lead the reader into the consideration of group-contribution methods, and both sections should serve to whet the appetite of the reader and lead him to pursue these topics further.

Although the method of presentation is obviously based upon the authors' experence in teaching, it is in the use of the *spiral technique* of presentation that the reviewer believes the student will encounter a problem. While the idea is appealing to one already familiar with thermodynamics, the use of this technique may well represent a sizeable hurdle for the beginning student, particularly in the initial portion of the book where possible difficulties are most noticeable. To illustrate this point, it is worth listing the topics presented in Chapter 1 (42 pages): traditional introductory material such as the definition of a system, the surroundings, heat, work, etc.; the definition of internal energy and a statement of the first law for a closed system; a definition of entropy (as an extensive property associated with thermal energy) and a brief statement of the second law; the definition of enthalpy, the Gibbs function, and the Helmholtz free energy, their general differential equations, and the Maxwell relations derived therefrom; the definitions of a property and of partial molar properties; the criteria of equilibrium and of a spontaneous process; the concept of lost work; the relation of entropy to probability; and an introduction to irreversible thermodynamics. While each of these topics is fully developed in subsequent chapters, at the end of this chapter the reader is overwhelmed.

The authors then continue with the same technique in Chapter 2 in which the relationships for various types of processes are presented for closed systems (in general and for ideal gases) before the first law is covered in detail in Chapter 3. It would seem that this portion of the book would be particularly difficult for a beginning student, leaving him lost, rather than with the intended clear sense of the purpose of the book.

Also to be noted is the statement on the jacket of the book that it "supplies the needs of the core curriculum undergraduate course in classical thermodynamics." While the book is well suited to a course for chemical engineering students, it does not actually supply the needs of a core course; indeed, the treatment of the first law should be supplemented in any case. In discussing the first law the authors present the equations for closed systems, steady-flow open systems, and nonsteady open systems, but the treatment of the latter is extremely brief. Actually, there are

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INFORMATION RETRIEVAL[*]

Oscillations of drops falling in a liquid field, Schroeder, R. R., and R. C. Kintner, **A.I.Ch.E. Journal, 11,** No. 1, p. 5 (January, 1965).

Key Words: Amplitude-2, 7, 8, Breakup-2, Drops-1, 9, Frequency-2, 7, 8, Mathematics-10, Oscillation-2, 7, 8, Photography-10, Velocity-1, 6, Viscosity-1, 5, 6, Vortex Trail-1, 6, Wall Effects-1, 5, 6.

Abstract: Frequencies and amplitudes of oscillations for nineteen pure systems were measured photographically. The period of oscillation was longer than that predicted by Lamb. The discrepancy was not due to wall effects, viscosity, or velocity of fall but to amplitude of oscillation. Modification of previous expressions included an amplitude function which could be experimental or empirical. Oscillations began near the peak velocity, and a vortex trail was necessary for them to take place. Oblate-prolate oscillations did not cause drop breakup, as all systems ceased to oscillate and wobbled randomly below maximum drop size. Oscillations do not decay with time.

Performance of gas bubble columns: volumetric liquid-phase mass transfer coefficient and gas holdup, Yoshida, Fumitake, and Kiyomi Akita, **A.I.Ch.E. Journal, 11,** No. 1, p. 9 (January, 1965).

Key Words: Volumetric Coefficient-7, Liquid-Phase Mass Transfer (Coefficient)-7, Gas Bubble Columns-10, Bubbling Gas-Liquid Contactor-10, Oxidation-8, Sodium Sulfite-1, Cupric Ions-4, Sodium Sulfate-9, Sulfite Oxidation-8, Desorption-8, Oxygen Desorption-8, Gas Holdup-7, Gas Velocity-6, Liquid Depth-6, Column Diameter-6, Gas Inlet Nozzle-6.

Abstract: The volumetric coefficient of liquid-phase mass transfer in four gas bubble columns of different diameters was studied by oxidation of sodium sulfite solutions with cupric ions as catalyst and desorption of oxygen from water and sodium sulfate solutions. The gas holdup was also determined with the same columns. Both the volumetric coefficient and the gas holdup vary mainly with the superficial gas velocity and are practically independent of the liquid depth and the diameter of the gas inlet nozzle. However, the effect of the column diameter on the volumetric coefficient cannot be neglected.

Heat transfer characteristics of concurrent gas-liquid flow in packed beds, Weekman, Vern W., Jr., and John E. Myers, **A.I.Ch.E. Journal, 11,** No. 1, p. 13 (January, 1965).

Key Words: Heat Transfer-7, 8, 9, Two Phase-6, 9, Flow-6, 9, Packed Bed-9, 10, Spheres-10, Pulses-6, 8, Gas Phase-5, Liquid Phase-5, Reactors-10, Velocity-6, Correlation-9, Penetration Theory-9.

Abstract: Radial heat transfer characteristics were studied for two-phase, concurrent flow of air and water through a 3-in. I.D. vertical column randomly packed with spheres. The air-water system gave as much as 400% greater heat transfer than previously published data for single-phase liquid flow through packed beds. The equivalent heat transfer coefficient h$_0$ varied from 50 to 1,100 B.t.u./(hr.) (°F.) (sq. ft.) as the gas and liquid rates were varied from 50 tol, 200 lb. air/(hr.) (sq. ft.) and 2,500 to 25,000 lb. water/(hr.) (sq. ft.), respectively. A radial transport model provided a satisfactory correlation for the heat transfer data. It was shown that penetration theory predicted the experimental heat transfer results to within ±20%. Transition from one flow regime to another gave significantly different heat transfer results.

A study of interstitial liquid flow in foam: Part I. Theoretical model and application to foam fractionation, Leonard, Ralph A., and Robert Lemlich, **A.I.Ch.E. Journal, 11,** No. 1, p. 18 (January, 1965).

Key Words: A. Foam-7,8, Theory-2,8, Model-2,8, Interstitial Flow-7,8, Drainage-7,8, Properties (Physical)-1,6, Surface Viscosity-1,6, Structure (Foam)-10, Capillary-10, Plateau Border-10, Momentum Balance-10, Difference Equations-10, Velocity Profiles-2,10, Numerical Solution-10, Digital Computer-10, Moving-, Stationary-, B. Flow Rates-2,9, Concentrations-2,9, Design-2,9, Column-9, Foam Fractionation-9, Separation-9, Reflux-9, Product-9, Waste-9, Pollution (Water)-9.

Abstract: A theoretical model for interstitial liquid flow in a stationary or moving foam was devised by relating the physical structure of the foam to the physical properties of the surfactant and the foam movement. This was accomplished through a differential momentum balance within a typical capillary (Plateau border) of noncircular cross section with finite surface viscosity at its boundaries. Velocity profiles were then calculated and integrated numerically for the randomly oriented capillaries so as to obtain the overall liquid flow through the foam in terms of the pertinent variables. Results are presented in a form suitable for estimating concentrations and flow rates of product and waste streams in foam fractionation.

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[*] For details on the use of these Key Words and the A.I.Ch.E. Information Retrieval Program, see **Chem. Eng. Progr.**, Vol. 60, No. 8, p. 88 (August, 1964). A free copy of this article may be obtained by sending a post card, with the words "Key Word Article" and your name and address (please print) to Publications Department, A.I.Ch.E., 345 East 47 St., N.Y. N.Y., 10017. Price quotations for volume quantities on request.

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If the boundary conditions

$$q = q_0 \text{ at } x = 0$$

and

$$q = q_L \text{ at } x = L$$

are used, it follows that

$$M = [q_L - q_0 e^{mL}]/[K(e^{-mL} - e^{mL})] \tag{2}$$

$$N = [q_L - q_0 e^{-mL}]/[K(e^{-mL} - e^{mL})] \tag{3}$$

For any fin i, then, it can be shown that

$$t_{0,i} = F_i q_{0,i} - G_i q_{L,i} + B_i \tag{4}$$

$$t_{L,i} = G_i q_{0,i} - F_i q_{L,i} + B_i \tag{5}$$

TWO FINS IN SERIES

When two fins 1 and 2 are connected

$$t_S - t_{0,1} = R_{S,1} q_{0,1};\ t_{L,1} - t_{0,2} = R_{1,2} q_{0,2};$$
$$q_{0,2} = f_{1,2} q_{L,1}; \text{ and } t_{L,2} - t_s = R_{2,s} q_{L,2}$$

so by using Equations (4) and (5) three simultaneous equations may be written:

$$t_{0,1} = F_1 q_{0,1} - G_1 q_{L,1} + B_1 = t_S - R_{S,1} q_{0,1} \tag{6}$$

$$t_{L,1} = G_1 q_{0,1} - F_1 q_{L,1} + B_1 = R_{1,2} q_{0,2} + t_{0,2} = R_{1,2} q_{0,2} + F_2 q_{0,2} - G_2 q_{L,2} + B_2 = (R_{1,2} + F_2) f_{1,2} q_{L,1} - G_2 q_{L,2} + B_2 \tag{7}$$

$$t_{L,2} = G_2 q_{0,2} - F_2 q_{L,2} + B_2 = G_2 f_{1,2} q_{L,1} - F_2 q_{L,2} + B_2 = R_{2,s} q_{L,2} + t_s \tag{8}$$

The unknowns in Equations (6), (7), and (8) are $q_{0,1}$, $q_{L,1}$, and $q_{L,2}$. By using determinants it is possible to show that

$$q_{L,1} = \{[(F_1 + R_{S,1})(B_2 - B_1) + G_1(B_1 - t_S)][F_2 + R_{2,s}] + G_2(F_1 + R_{S,1})(t_s - B_2)\}/\{\{-[F_1 + R_{S,1}][F_1 + f_{1,2}(F_2 + R_{1,2})] + G_1^2\}\{F_2 + R_{2,s}\} + f_{1,2} G_2^2 (F_1 + R_{S,1})\} \tag{9}$$

[Although Equation (9) and the other solution equations may appear formidable, in most cases many of the terms are either zero or unity.] $q_{0,1}$ can then be found from Equation (6), $t_{L,1}$ and $q_{L,2}$ from Equation (7), and $t_{L,2}$ from Equation (8).

THREE FINS IN SERIES

When three fins 1, 2, and 3 are con-

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nected, simultaneous Equations (6) and (7) are still valid. Since

$$t_{L,2} - t_{0,3} = R_{2,3}q_{0,3};\ q_{0,3} = f_{2,3}q_{L,2};$$

$$\text{and} \qquad t_{L,3} - t_s = R_{3,s}q_{L,3}$$

Equations (4) and (5) may be used to write two additional simultaneous equations:

$$t_{L,2} = G_2f_{1,2}q_{L,1} - F_2q_{L,2} + B_2 = R_{2,3}q_{0,3} + t_{0,3} = R_{2,3}q_{0,3} + F_3q_{0,3} - G_3q_{L,3} + B_3 = (R_{2,3} + F_3)f_{2,3}q_{L,2} - G_3q_{L,3} + B_3 \qquad (10)$$

$$t_{L,3} = G_3q_{0,3} - F_3q_{L,3} + B_3 = G_3f_{2,3}q_{L,2} - F_3q_{L,3} + B_3 = R_{3,s}q_{L,3} + t_s \qquad (11)$$

The unknowns in Equations (6), (7), (10), and (11) are $q_{0,1}$, $q_{L,1}$, $q_{L,2}$, and $q_{L,3}$. Use of determinants gives

$$q_{L,1} = \{\{(F_1 + R_{S,1})(B_2 - B_1) + G_1(B_1 - t_S)\}\{[F_2 + f_{2,3}(F_3 + R_{2,3})][F_3 + R_{3,s})] - f_{2,3}G_3^2\} + G_2(F_1 + R_{S,1})[(B_3 - B_2)(F_3 + R_{3,s}) + G_3(t_s - B_3)]\}/\{\{-[F_1 + R_{S,1}][F_1 + f_{1,2}(F_2 + R_{1,2})] + G_1^2\}\{[F_2 + f_{2,3}(F_3 + R_{2,3})][F_3 + R_{3,s}] - f_{2,3}G_3^2\} + f_{1,2}G_2^2(F_1 + R_{S,1})(F_3 + R_{3,s})\} \qquad (12)$$

$q_{0,1}$ can then be found from Equation (6), $t_{L,1}$ and $q_{L,2}$ from Equation (7), $t_{L,2}$ and $q_{L,3}$ from Equation (10), and $t_{L,3}$ from Equation (11).

FOUR FINS IN SERIES

When four fins 1, 2, 3, and 4 are connected, simultaneous Equations (6), (7), and (10) are still valid. Since

$$t_{L,3} - t_{0,4} = R_{3,4}q_{0,4};\ q_{0,4} = f_{3,4}q_{L,3};$$

$$\text{and} \qquad t_{L,4} - t_s = R_{4,s}q_{L,4}$$

Equations (4) and (5) may be used to write two additional simultaneous equations:

$$t_{L,3} = G_3f_{2,3}q_{L,2} - F_3q_{L,3} + B_3 = R_{3,4}q_{0,4} + t_{0,4} = R_{3,4}q_{0,4} + F_4q_{0,4} - G_4q_{L,4} + B_4 = (R_{3,4} + F_4)f_{3,4}q_{L,3} - G_4q_{L,4} + B_4 \qquad (13)$$

$$t_{L,4} = G_4q_{0,4} - F_4q_{L,4} + B_4 = G_4f_{3,4}q_{L,3} - F_4q_{L,4} + B_4 = R_{4,s}q_{L,4} + t_s \qquad (14)$$

The unknowns in Equations (6), (7), (10), (13), and (14) are $q_{0,1}$, $q_{L,1}$, $q_{L,2}$, $q_{L,3}$, and $q_{L,4}$. Use of determinants gives

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only six examples in the chapter dealing with the first law; five are steady-flow systems, and the sixth, while dealing with nonsteady flow, is a simple mass balance, not an application of the first law. Nineteen pages at the end of this chapter are devoted to gas dynamics, but no qualitative discussion of the behavior of a fluid in a nozzle is given, the significance of stagnation properties is not emphasized, and shock waves are discussed with no mention of the Fanno or Rayleigh lines. The authors stated their intention of de-emphasizing equipment, but the presentation of gas dynamics would benefit by the discussion of the behavior of a fluid in a nozzle, for it is from this discussion that the student gains a qualitative understanding of the necessary existence of a shock wave.

In addition to the virtual omission of the nonsteady flow system from the first law presentation, one other point detracts from the value of this book for a core course. Three excellent chapters (115 pages) are included which deal with solutions and phase equilibrium on multicomponent systems. The authors stress the use of the Gibbs function as the criterion for the existence of a stable phase, and the conditions for phase equilibrium are clearly illustrated through the use of free-energy diagrams showing the equality of the Gibbs function in phase equilibrium and the concept of the Gibbs function as a potential in nonequilibrium situations. This discussion is extremely valuable for the chemical engineer, however, the emphasis on separation processes here would appear to be a defect if the book is to be used in a core course.

Two minor omissions which are certainly not specific to this book are worth mention. First, the Gibbs-Duhem equation is first presented without the terms which account for the effects of temperature and pressure, but the limitations in applying this equation to binary phase equilibrium are not clearly stated. Second, there is a section dealing with simultaneous reaction equilibria, but thre is no discussion of the method to be used to determine the number of reactions to be considered for a system with a given number of species. These two points would certainly need to be clarified by an instructor.

Clarification would also seem warranted on two further points. In a short section on enthalpy-concentration diagrams and their application, the authors seem to have sacrificed clarity for the sake of brevity. Further, a property is defined in terms of a line integral and the use of Green's theorem. While this definition is certainly completely rigorous, it is probably beyond the mathematical capability of a normal college junior.

In conclusion, it should be noted again that the book has many good points, and that the stress on topics of interest to chemical engineers makes the book suitable for a thermodynamics course for this group. However, this same stress and the treatment of the first law would seem to make the book unsuitable for a core curriculum course. In addition, it seems to the reviewer that the *spiral technique*, while admirable in principle, would cause more problems than it solves.

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INFORMATION RETRIEVAL

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A study of interstitial liquid flow in foam; Part II. Experimental verification and observations, Leonard, Ralph A., and Robert Lemlich, **A.I.Ch.E. Journal, 11,** No. 1, p. 25 (January, 1965).

Key Words: A. Foam-8, Experiments-1, Drainage-2, 8, Bubbles-8, Variables-1, 6, Fractionation-2, 8, Columns-10. B. Experimental Results-8, Model-9, Theory-9, Agreement-, Part I-.

Abstract: Experimental results obtained over a wide range of variables from three different foam fractionation columns support the theory developed in Part I.

Turbulence characteristics of liquids in pipe flow, Martin, Godfrey Q., and Lennart N. Johanson, **A.I.Ch.E. Journal, 11,** No. 1, p. 29 (January, 1965).

Key Words: Water-5, Pipe Flow-5, Velocity-6, Turbulence-7, Relative Intensity-7, Integral Scale-7, Eddy Diffusivity-7, Hit-Film Anemometer-10, Eulerian-, Lagrangian-0, Statistical-0.

Abstract: The relative intensity and Eulerian integral scale of turbulence have been measured, with hot-film anemometry techniques, at the center of a 6-in. pipe through which water was pumped with velocities of 0.50 to 4.0 ft./sec. $N_{Re} =$ 19,000 to 160,000). Eulerian integral scale of turbulence data have been very meager in this Reynolds number region, owing perhaps to the low frequency of the velocity fluctuations encountered. Previous relative intensity of turbulence data have been obtained with gases as the fluid system.

With the diffusion model of G. I. Taylor used, the results obtained have been correlated with the Reynolds number of flow and also with eddy diffusivities and Lagrangian turbulence parameters reported by other investigators.

Adsorption rate studies—interaction of diffusion and surface processes, Masamune, Shinobu, and J. M. Smith, **A.I.Ch.E. Journal, 11,** No. 1, p. 34 (January, 1965).

Key Words: Adsorption-8, Time-6, Concentration-3, Intraparticle Diffusion-9, Surface Reaction-9, External Diffusion-9, Fixed Bed-10, Adsorbent-4, Rate-8, Reactor-10.

Abstract: Solutions are presented for transient adsorption in a fixed bed when one, two, or all of the following processes affect the overall rate: surface adsorption, intraparticle diffusion, and external diffusion. These results, based upon a first-order kinetic equation, are given in the same form for all cases, each solution being in terms of the appropriate dimensionless groups. The solutions are illustrated numerically for two special cases: intraparticle diffusion and surface adsorption together control the rate, and surface adsorption alone is the controlling process.

Adsorption of ethyl alcohol on silica gel, Masamune, Shinobu, and J. M. Smith, **A.I.Ch.E. Journal, 11,** No. 1, p. 41 (January, 1965).

Key Words: Ethyl Alcohol-1, Silica Gel-4, 5, Time-6, Particle Size-6, Temperature-6, Adsorption-8, Diffusion-9, Surface Migration-9, Fixed Bed-10, Reactor-10.

Abstract: Adsorption data, in the form of breakthrough curves, were measured for ethyl alcohol on beds of silica gel at 90° to 155°C. For the smallest particle size of gel and at the lower temperatures the rate of adsorption was determined by the surface adsorption process. At other conditions both surface adsorption and intraparticle diffusion determined the rate. Analysis of the diffusivity results indicated that surface migration on the pore walls was primary contribution to diffusion.

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$$q_{L,1} = \{\{(F_1 + R_{S,1})(B_2 - B_1) + G_1(B_1 - t_S)\}\{\{F_2 + f_{2,3}(F_3 + R_{2,3})\}\{[F_3 + f_{3,4}(F_4 + R_{3,4})][F_4 + R_{4,s}] - f_{3,4}G_4^2\} - f_{2,3}G_3^2(F_4 + R_{4,s})\} + G_2\{F_1 + R_{S,1}\}\{\{B_3 - B_2\}\{[F_3 + f_{3,4}(F_4 + R_{3,4})][F_4 + R_{4,s}] - f_{3,4}G_4^2\} + G_3[(B_4 - B_3)(F_4 - R_{4,s}) + G_4(t_s - B_4)]\}\}/\{\{-[F_1 + R_{S,1}][F_1 + f_{1,2}(F_2 + R_{1,2})] + G_1^2\}\{\{F_2 + f_{2,3}(F_3 + R_{2,3})\}\{[F_3 + f_{3,4}(F_4 + R_{3,4})][F_4 + R_{4,s}] - f_{3,4}G_4^2\} - f_{2,3}G_3^2(F_4 + R_{4,s})\} + f_{1,2}G_2^2\{F_1 + R_{S,1}\}\{[F_3 + f_{3,4}(F_4 + R_{3,4})][F_4 + R_{4,s}] - f_{3,4}G_4^2\}\} \quad (15)$$

$q_{0,1}$ can then be found from Equation (6), $t_{L,1}$ and $q_{L,2}$ from Equation (7), $t_{L,2}$ and $q_{L,3}$ from Equation (10), $t_{L,3}$ and $q_{L,4}$ from Equation (13), and $t_{L,4}$ from Equation (14).

PLATE FIN WITH DIFFERENT CONDITIONS AT EACH SURFACE

If the two surfaces A and B of a plate fin have different conditions, the fin differential equation *(1)* becomes

$$kA\frac{d^2t}{dx^2} = h_A C_A(t - t_{e,A}) + h_B C_B(t - t_{e,B}) - q'''A \quad (16)$$

or

$$\frac{d^2t}{dx^2} = \left(\frac{h_A C_A + h_B C_B}{kA}\right)t - \frac{h_A C_A t_{e,A} + h_B C_B t_{e,B}}{kA} - \frac{q'''}{k} \quad (17)$$

When one makes the following substitutions

$$\overline{hC} = h_A C_A + h_B C_B \quad (18)$$

$$\overline{t_e} = (h_A C_A t_{e,A} + h_B C_B t_{e,B})/(h_A C_A + h_B C_B) \quad (19)$$

$$\overline{m}^2 = \overline{hC}/kA \quad (20)$$

$$\overline{\theta} = t - \overline{t_e} \quad (21)$$

Equation (17) becomes

$$\overline{\theta} = \overline{M}e^{-\overline{m}x} + \overline{N}e^{\overline{m}x} + [q'''/(k\overline{m}^2)] \quad (22)$$

Owing to the similarity of Equations (22) and (1), all results given earlier can also be used for plate fins with different surface conditions, provided the following substitutions are used:

$$hC = \overline{hC} \text{ from Equation (18)} \quad (23)$$

$$t_e = \overline{t_e} \text{ from Equation (19)} \quad (24)$$

NOTATION

A = cross-sectional area, sq. ft.

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B = $(q''' A/hC) + t_e$, F
C = perimeter (or surface area per unit axial distance), ft.
f = fraction of heat leaving downstream end of one fin which enters upstream end of next fin (usually unity), dimensionless
F = $(e^{mL} + e^{-mL})/[K(e^{mL} - e^{-mL})] = (\coth mL)/K$, hr. $F\,B^{-1}$
G = $2/[K(e^{mL} - e^{-mL})] = 1/K \sinh mL$, hr. $F\,B^{-1}$
h = film coefficient or surface conductance, B hr.$^{-1}$ ft.$^{-2}$ F^{-1}
k = thermal conductivity, B hr.$^{-1}$ ft.$^{-1}$ F^{-1}
K = $(kAhC)^{1/2}$, B hr.$^{-1}$ F^{-1}
L = axial length of fin, ft.
m = $(hC/kA)^{1/2}$, ft.$^{-1}$
q = axial heat flow rate, B hr.$^{-1}$
q''' = internal heat generation rate per unit volume, B hr.$^{-1}$ ft.$^{-3}$
R = thermal resistance (often zero) between downstream end of one fin and upstream end of next fin, between source and upstream end of first fin, or between downstream end of last fin and sink, hr. ft. $F\,B^{-1}$
t = temperature, F
x = distance in axial direction, ft.
θ = temperature difference between fin and surroundings, F

Subscripts

e = environment or surroundings
i = individual fin
L = downstream end of fin
N = total number of fins
S = source
s = sink
0 = upstream end of fin
1, 2, 3, or 4 = fin number

LITERATURE CITED

1. Jakob, Max, "Heat Transfer," 1 ed., p. 245, Wiley, New York (1949).